Global statistical model calculations and the role of isospin

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Abstract. An improved code for the calculation of astrophysical reaction rates in the statistical model is presented. It includes the possibility to study isospin effects. Such effects heavily affect rates involving self–conjugate nuclei and may also be found in reactions on other intermediate and heavy targets.

1. Introduction

The investigation of explosive nuclear burning in astrophysical environments is a challenge for both theoretical and experimental nuclear physicists. Highly unstable nuclei are produced in such processes which again can be targets for subsequent reactions. Cross sections and astrophysical reaction rates for a large number of nuclei are required to perform complete network calculations which take into account all possible reaction links and do not postulate a priori simplifications.

The majority of reactions can be described in the framework of the statistical model (compound nucleus mechanism, Hauser–Feshbach approach, HF) [1], provided that the level density of the compound nucleus is sufficiently large in the contributing energy window [2]. In astrophysical applications usually different aspects are emphasized than in pure nuclear physics investigations. Many of the latter in this long and well established field were focused on specific reactions, where all or most "ingredients", like optical potentials for particle transmission coefficients, level densities, resonance energies and widths of giant resonances to be implemented in predicting E1 and M1 γ–transitions, were deduced from experiments. As long as the statistical model prerequisites are met, this will produce highly accurate cross sections. For the majority of nuclei in astrophysical applications such information is not available. The real challenge is thus not the well–established statistical model, but rather to provide all these necessary ingredients in as reliable a way as possible, also for nuclei where none of such information is available.

In Section 2, an improved code for the calculation of astrophysical reaction rates in the statistical model is briefly presented. It includes the possibility of studying isospin effects. The latter are further discussed in Section 3.

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2. The code NON–SMOKER

Based on the well–known code SMOKER [3], an improved code for the prediction of astrophysical cross sections and reaction rates in the statistical model has been developed. The current status of the new code NON–SMOKER is described in the following.

The final quantities entering the expression for the cross section in the statistical model [1] are the averaged transmission coefficients. They do not reflect a resonance behavior but rather describe absorption via an imaginary part in the (optical) nucleon–nucleus potential [4]. In astrophysics, usually reactions induced by light projectiles (neutrons, protons, α particles) are most important. Global optical potentials are quite well defined for neutrons and protons. It was shown [3, 5, 6] that the best fit of s–wave neutron strength functions is obtained with the optical potential by [7], based on microscopic infinite nuclear matter calculations for a given density, applied with a local density approximation. It includes corrections of the imaginary part [8, 9]. A similar description is used for protons.

Optical potentials for α particles are treated in the folding approach [10], with a parametrized mass– and energy–dependence of the real volume integral [11]. The mass– and energy–dependence of the imaginary potential is parametrized according to [12] and additionally includes microscopic and deformation information [11].

Deformed nuclei are treated by an effective spherical potential of equal volume [3, 6]. For a detailed description of the formalism used to calculate E1 and M1 γ–transmission coefficients and the inclusion of width fluctuation corrections [13], see [2] and references therein.

The level density treatment has been recently improved [2]. However, the problem of the parity distribution at low energies remained. The new code includes a modified version of the description [2], accounting for non–evenly distributed parities at low energies, based on most recent findings within the framework of the shell model Monte Carlo method [14].

Additionally, the included data set of experimental level information (excitation energies, spins, parities) has been updated [15], as well as the experimental nuclear masses [16]. These data bases are continuously updated. For theoretical masses, there is a choice between different mass models (e.g. by Hilf *et al* [17], FRDM [18], ETFSI [19]), of which currently the FRDM is favored. Microscopic information needed for the calculation of level densities and α+nucleus potentials are also taken from the FRDM, as well as experimentally unknown ground state spins [20].

Finally, isobaric analog states $T^{>} = T^{<} + 1 = T^{\mathrm{g.s.}} + 1$ are explicitly considered in the new code. This will be discussed in the next section.

3. Inclusion of isospin

The original HF equation [1] implicitly assumes complete isospin mixing but can be generalized to explicitly treat the contributions of the dense background states with isospin $T^{<} = T^{\mathrm{g.s.}}$ and the isobaric analog states with $T^{>} = T^{<} + 1$ [21, 22, 23, 24]. In reality, compound nucleus states do not have unique isospin and for that reason an isospin mixing parameter $\mu\downarrow$ was introduced [21], which is the fraction of the width of $T^{>}$ states leading to $T^{<}$ transitions. For complete isospin mixing $\mu\downarrow= 1$, for pure $T^{<}$

states $\mu\downarrow=0$. In the case of overlapping resonances for each involved isospin, $\mu\downarrow$ is directly related to the level densities $\rho^<$ and $\rho^>$, respectively. Isolated resonances can also be included via their internal spreading width $\Gamma^\downarrow$ and a bridging formula was derived to cover both regimes [25].

In order to determine the mixing parameter $\mu\downarrow=\mu\downarrow(E)$, experimental information for excitation energies of $T^>$ levels is used where available [15, 26] in the code NON–SMOKER. Experimental values for spreading widths are also tabulated [24, 26]. Similarly to the standard treatment for the $T^<$ states, a level density description [2] is invoked above the last experimentally known $T^>$ level. Since the $T^>$ states in a nucleus (Z,N) are part of multiplet, they can be approximated by the levels (and level density) of the nucleus $(Z-1,N+1)$, only shifted by a certain energy E_d. This displacement energy can be calculated [27] and it is dominated by the Coulomb displacement energy $E_\mathrm{d}=E_\mathrm{d}^\mathrm{Coul}+\epsilon$.

The inclusion of the explicit treatment of isospin has two major effects on statistical cross section calculations in astrophysics: the suppression of γ–widths for reactions involving self–conjugate nuclei and the suppression of the neutron emission in proton–induced reactions. Non–statistical effects, i.e. the appearance of isobaric analog resonances, are included in the treatment of the mixing parameter $\mu\downarrow$ [25] but will not be further discussed here.

3.1. γ–Widths

The isospin selection rule for $E1$ transitions is $\Delta T=0,1$ with transitions $0\rightarrow 0$ being forbidden [28]. An approximate suppression rule for $\Delta T=0$ transitions in self–conjugate nuclei can also be derived for $M1$ transitions [28].

In the case of (α,γ) reactions on targets with $N=Z$, the cross sections will be heavily suppressed because $T=1$ states cannot be populated due to isospin conservation. A suppression will also be found for capture reactions leading into self–conjugate nuclei, although somewhat less pronounced because $T=1$ states can be populated according to the isospin coupling coefficients.

In previous reaction rate calculations [29, 6, 30] the suppression of the γ–widths was treated completely phenomenologically by dividing by quite uncertain factors of 5 and 2, for (α,γ) reactions and nucleon capture reactions, respectively. In the new code NON–SMOKER, the appropriate γ–widths are automatically obtained, by explicitly accounting for population and decay of $T^<$ and $T^>$ states, and considering isospin mixing by the parameter $\mu\downarrow$.

3.2. Competition cusps in proton–induced reactions

Assuming incomplete isospin mixing, the strength of the neutron channel will be suppressed in comparison to the proton channel in reactions p+target [21, 23, 24]. This leads to a smaller cross section for (p,n) reactions and an increase in the cross section of (p,γ) reactions above the neutron threshold, as compared to calculations neglecting isospin (i.e. implicitly assuming complete isospin mixing with $\mu\downarrow=1$).

The isospin mixing parameter was varied in the theoretical investigation of a $^{51}\mathrm{V}(\mathrm{p},\gamma)^{52}\mathrm{Cr}$ experiment [31]. It was found [31] that complete isospin mixing closely

reproduced the measured cross sections when width fluctuation corrections were considered. Width fluctuation corrections [32] affect the (p,γ) cross sections above as well as below the neutron threshold, whereas incomplete isospin mixing only reduces the cross sections above the threshold. Thus, the two corrections can be discriminated. Mainly from that result, it was concluded that — contrary to width fluctuation corrections — isospin can be neglected.

However, a closer investigation of the $T^>$ levels in ^{52}Cr (using [25] and [2]) shows that isospin mixing should be rather complete already at the neutron threshold (since the first $T^>$ state is almost 1 MeV below the threshold [15]). This is also true for lighter targets. Nevertheless, for reactions on more heavy nuclei ($Z > 30$) the neutron and proton threshold, respectively, will still be in a region of incomplete isospin mixing and therefore isospin effects should be detectable there. This effect, however, does not play such an important role in the calculation of astrophysical reaction rates as the suppression of the γ–width described in the previous chapter, because of the averaging over an energy range (the Gamow window) in the calculation of the rate.

4. Conclusion

The new code NON–SMOKER makes use of the latest set of descriptions for the calculation of the nuclear properties needed to reliably predict astrophysical reaction rates, such as masses, level densities, nucleon– and α–potentials, GDR energies and widths, width fluctuation corrections. Additionally, the possibility of studying isospin effects has been included. This also leads to a more fundamental treatment of the γ–width suppression for compound nuclei with $T^{\text{g.s.}} = T^< = 0$.

Nevertheless, more experimental data are needed to check and further improve current parametrizations. Especially investigations over a large mass range would prove useful to fill in gaps in the knowledge of the nuclear structure of many isotopes and to construct more powerful parameter systematics. Such investigations should include neutron–, proton– and α–strength functions, as well as radiative widths, and charged particle scattering and reaction cross sections for *stable* and unstable isotopes. More capture data with self–conjugate final nuclei would also be highly desireable.

This information can be used to make future large–scale statistical model calculations even more accurate.

References

[1] Hauser W and Feshbach H 1952 *Phys. Rev.* **87** 366

[2] Rauscher T, Thielemann F–K and Kratz K–L 1997 *Phys. Rev.* C **56** 1613

[3] Thielemann F–K, Arnould M and Truran J W 1987 *Advances in Nuclear Astrophysics*, ed E Vangioni-Flam (Gif sur Yvette: Editions Frontière) p 525

[4] Mahaux C and Weidenmüller H A 1979 *Ann. Rev. Part. Nucl. Sci.* **29** 1

[5] Thielemann F–K, Metzinger J and Klapdor H V 1983 *Z. Phys.* A **309** 301

[6] Cowan J J, Thielemann F–K and Truran J W 1991 *Phys. Rep.* **208** 267

[7] Jeukenne J P, Lejeune A and Mahaux C 1977 *Phys. Rev.* C **16** 80

[8] Fantoni S, Friman B L and Pandharipande V R 1981 *Phys. Rev. Lett.* **48** 1089

[9] Mahaux C 1982 *Phys. Rev.* C **82** 1848

[10] Satchler G R and Love W G 1979 *Phys. Rep.* **55** 183

[11] Rauscher T 1998 *"Nuclear Astrophysics", Proc. Int. Workshop XXVI on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg* eds J Wambach *et al* (Darmstadt: Gesellschaft für Schwerionenforschung) in press

[12] Brown G E and Rho M 1981 *Nucl. Phys.* **A372** 397

[13] Tepel J W, Hoffmann H M and Weidenmüller H A 1974 *Phys. Lett.* **49B** 1

[14] Nakada H and Alhassid Y 1997 *Phys. Rev. Lett.* **79** 2939

[15] *Data base ENSDF*, NNDC, Brookhaven National Laboratory, Upton, USA

[16] Audi G and Wapstra A H 1995 *Nucl. Phys.* **A595** 409

[17] Hilf E R, von Groote H and Takahashi K 1976 *Proc. Third Int. Conf. on Nuclei far from Stability* (Geneva: CERN **76**-13) p 142

[18] Möller P, Nix J R, Myers W D, and Swiatecki W J 1995 *At. Data Nucl. Data Tables* **59** 185

[19] Aboussir Y, Pearson J M, Dutta A K, and Tondeur F 1995 *At. Data Nucl. Data Tables* **61** 127

[20] Möller P, Nix J R and Kratz K–L 1997 *At. Data Nucl. Data Tables* **66** 131

[21] Grimes S M, Anderson J D, Kerman A K, and Wong C 1972 *Phys. Rev.* C **5** 85

[22] Harney H L, Weidenmüller H A and Richter A 1977 *Phys. Rev.* C **16** 1774

[23] Sargood D G 1982 *Phys. Rep.* **93** 61

[24] Harney H L, Richter A and Weidenmüller H A 1986 *Rev. Mod. Phys.* **58** 607

[25] Lane A M 1978 *Phys. Rev.* C **18** 1525

[26] Reiter J and Harney H L 1990 *Z. Phys.* A **337** 121

[27] Auerbach N, Hüfner J, Kerman A K, and Shakin C M 1972 *Rev. Mod. Phys.* **44** 48

[28] Jones G A 1987 *The Properties of Nuclei, Second Edition* (Oxford: Oxford University Press)

[29] Woosley S E, Fowler W A, Holmes J A, and Zimmerman B A 1978 *At. Data Nucl. Data Tables* **22** 371

[30] Schatz H *et al* 1998 *Phys. Rep.* **294** 167

[31] Zyskind J L, Barnes C A, Davidson J M, Fowler W A, Marrs R E, and Shapiro M H 1980 *Nucl. Phys.* **A343** 295

[32] Verbaatschot J J M, Weidenmüller H A and Zirnbauer M R 1984 *Phys. Rep.* **129** 367